

Research report

Actively tracking ‘passive’ stability in a ball bouncing task

Aymar de Rugy^{a,*}, Kunlin Wei^a, Hermann Müller^{a,b}, Dagmar Sternad^a

^aDepartment of Kinesiology, 266 Recreation Building, The Pennsylvania State University, University Park, PA 16802, USA

^bInstitute for Sport Science, University of the Saarland, Saarbrücken, Germany

Accepted 2 May 2003

Abstract

This study investigates the control involved in a task where subjects rhythmically bounce a ball with a hand-held racket as regularly as possible to a prescribed amplitude. Stability analyses of a kinematic model of the ball–racket system revealed that dynamically stable solutions exist if the racket hits the ball in its decelerating upward movement phase. Such solutions are resistant to small perturbations obviating explicit error corrections. Previous studies reported that subjects’ performance was consistent with this ‘passive’ stability. However, some ‘active’ control is needed to attune to this passive stability. The present study investigates this control by confronting subjects with perturbations where stable behavior cannot be maintained solely from passive stability. Six subjects performed rhythmic ball bouncing in a virtual reality set-up with and without perturbations. In the perturbation trials the coefficient of restitution of the ball–racket contact was changed at every fifth contact leading to unexpected ball amplitudes. The perturbations were compensated for within 2–3 bouncing cycles such that ball amplitudes decreased to initial values. Passive stability was reestablished as indicated by negative racket acceleration. Results revealed that an adjustment of the racket period ensured that the impacts occurred at a phase associated with passive stability. These findings were implemented in a model consisting of a neural oscillator that drives a mechanical actuator (forearm holding the racket) to bounce the ball. Following the perturbation, the oscillator’s period is adjusted based on the perceived ball velocity after impact. Simulation results reproduced the major aspects of the experimental results.

© 2003 Elsevier B.V. All rights reserved.

Theme: Motor systems and sensorimotor integration

Topic: Control of posture and movement

Keywords: Ball bouncing; Rhythmic movement; Dynamical stability; Neural oscillator

1. Introduction

The task of ‘juggling’ or bouncing a ball cyclically on a racket has received considerable attention in recent years in both the robotics and the motor control literature [1–4,6,9,16–20]. The reason for this interest is that this task constitutes an exemplary case where an actor, or more precisely an end-effector, i.e., the racket, interacts with an object in the environment, i.e., the ball. The movement of the racket and the racket–ball contact determines the flight trajectory of the ball to which, in turn, the racket has to synchronize again. The classical approach in control theory would suggest that the trajectories of the racket need to be planned and monitored based on feedback from the ball’s

trajectory. Along this line, Koditschek and co-workers designed a robot actuator capable of bouncing ball in three dimensions [6,16]. The racket movements were controlled by the ‘mirror algorithm’, which matched the actuator’s velocity to the velocity of the ball with opposite sign (with a gain), i.e., ball and actuator were tightly coupled at every moment in time.

Further analysis of the bouncing-ball system revealed that such continuous closed-loop control of the racket trajectory is not necessary as only the contact event has an effect on the ball’s trajectory. Further, the work of Wood and Byrne [22], Holmes [8] and Guckenheimer and Holmes [7] showed that a ball bouncing on a periodically driven planar surface exhibits dynamically stable solutions, i.e., stable performance is obtained in an open-loop fashion without continuous feedback from the ball trajectory (see also Ref. [21]). Schaal et al. [17] extended these analyses,

*Corresponding author.

E-mail address: aug3@psu.edu (A. de Rugy).

which were initially valid for a table vibrating with a small amplitude, to the range of motion involved when human actors bounce a ball on a racket. Stability analyses revealed that stable bouncing can be achieved with an arbitrary periodic motion of the racket, provided that the successive impacts occur during upward movements of the racket that display negative acceleration. In the context of this study, we refer to this mode of performance as ‘passive stability’, since no explicit corrective control of the racket with respect to the ball’s trajectory is required.

Fig. 1 displays the periodic motion of a racket together with ball trajectories that were generated by using equations for ballistic flight and elastic impact. These trajectories, which were generated by the model described in the discussion below, illustrate how a decelerating upward movement of the racket at impact gives rise to passive stability. Three simulations are shown where for two of them a small perturbation was applied on the second impact shown. One of these perturbations (bold curve) results in a lower ball amplitude than that of the preceding unperturbed ball trajectory. As there is no modulation of the racket movement in the passively stable mode, the ball–racket contact of the succeeding contact occurs earlier than in the preceding contact. Since racket acceleration is negative in this segment of the trajectory, an earlier contact is associated with a higher racket velocity, which in turn gives rise to a higher ball amplitude in the next cycle. The opposite chain of events holds for perturbations with a higher than average ball amplitude. As a result, these small perturbations die out and converge back to the initial unperturbed amplitudes after few contacts. Formal stability analyses of the ball-bouncing map can be found in Ref. [19].

Bouncing the ball with ball–racket contacts in this decelerating racket movement phase, i.e., with passive stability, thus represents an alternative to the explicit specification of a desired trajectory with continuous coupling between the racket and ball movements. Sternad and co-workers showed in different task variations that subjects

perform ball bouncing with negative acceleration of the racket at impact, indicative of a strategy that exploits passive stability to maintain stable bouncing [9,17,19,20]. This can be interpreted that such open-loop control has less computational demands on the controller and is thus more efficient. However, these results do not exclude that other control strategies are involved over and above exploiting stability properties. In fact, Sternad et al. [20] showed that performance was more variable when visual information was excluded and when only haptic information about successive impacts was available. Furthermore, in order to establish this passive stability when starting periodic bouncing movement, the racket has to be controlled with respect to the ball. Active control is also expected to prevail when large perturbations are applied. To throw light on such control mechanisms the present study introduced sufficiently large perturbations that were outside the realm of passive stabilization. A virtual reality set-up allowed us to introduce such perturbations during a ball bouncing performance. Our aim was to examine how subjects recover from these perturbations. Subsequently, a model is developed which consists of a neural oscillator that drives a mechanical actuator (forearm holding the racket) that bounces a ball. This model exhibits dynamical stability but also includes a control algorithm that counteracts large perturbations. The model is shown to replicate the major experimental results.

2. Materials and methods

2.1. Participants

Six volunteers participated in this experiment. Their age ranged between 25 and 43 years (four male, two female). All were right-handed and used their preferred right hand to bounce the ball with the racket. The participants were informed about the experimental procedure and signed the consent form in compliance with the Regulatory Committee of the Pennsylvania State University.

2.2. Virtual reality apparatus and data collection

In the virtual reality set-up the subject manipulated a real tennis racket in front of a large screen (1.4 m wide and 1.5 m high) onto which the visual display was projected (Fig. 2). The subject stood upright at a distance of 1.5 m from the screen and held the racket horizontally at a comfortable height. A rigid rod with three hinge joints and one swivel joint was attached to the racket surface and ran through a noose that rotated a wheel by its vertical motion. The material of the rod minimized friction. Due to the joints the racket could be moved and tilted freely in three dimensions. The revolutions of this wheel were measured by an optical encoder. The digital signal from the optical encoder was transformed by a digital board and sent online

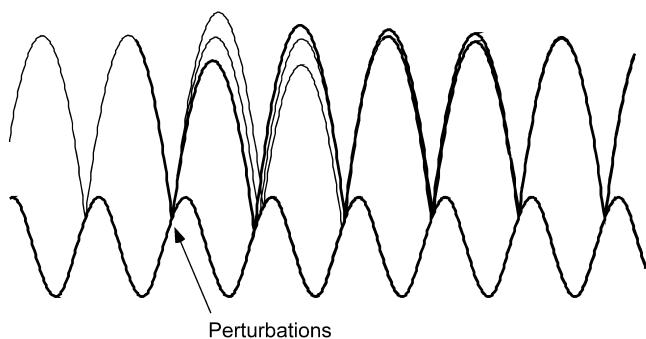


Fig. 1. Illustration of ‘passive’ stability associated with negative acceleration of the racket at impact. After small perturbations applied at the second contact to two of the three ball trajectories, the trajectories converge back to a stable bouncing pattern, without any additional modulation of the racket trajectory (see text).

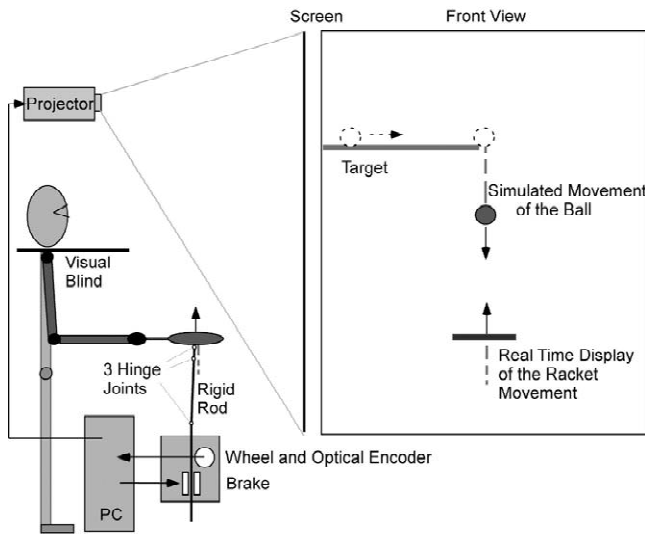


Fig. 2. Virtual reality set-up.

to the computer that controlled the experimental set-up. The movements of the racket were projected onto the screen. Displacement of the virtual racket on the screen corresponded to the real racket movements amplified by a factor of 3.5. As a result, subjects had to move the racket 3.5 time less in our apparatus than in reality to give rise to the same ball trajectory. This gain was chosen to keep the real racket's amplitude and velocity at a moderate level. Higher velocities introduced measurement inaccuracies due to friction and vibrations. Subjects adapted to the virtual set-up very easily and were not aware of this discrepancy between the real and virtual racket movements. A sheet of cardboard was attached to the subject's neck to eliminate vision of the physical racket that the subject manipulated. Thereby, the subject could only see her/his racket movements on the screen. In order to simulate the mechanical contact between the racket and the ball, a mechanical brake was attached to the rod. The brake was applied at each impact for a duration of 30 ms. The electronic delay due to computations had a duration of only one sampling interval (3 ms). The mechanical delay between the control signal and the onset of the brake was approximately 20 ms. Subjects did not notice this delays and perceived the visual and the haptic contact information as simultaneous. The force developed by this brake was adjusted to that produced by a tennis ball falling on the racket. The force was approximately constant for all contacts.

Custom-written software computed online the ball trajectories based on the measured racket movements and the contact parameters of ball and racket. The simulated ball trajectories were projected onto the screen so that the subject only interacted virtually with the ball (Open GL Graphics). The visual display was calculated every 3 ms, based on the acquisition rate of 333 Hz. This visual display was projected to the screen with an update rate of 75 Hz. The ball's trajectory was calculated using the equations of

ballistic flight and elastic impact. Whenever the ball was in the air, its vertical motion was only influenced by gravity and its ballistic flight was described by:

$$x_B = -1/2gt^2 + \dot{x}_{B,0}t + x_{B,0} \quad (1)$$

where x_B is the vertical position of the ball, g is gravity, and $x_{B,0}$ and $\dot{x}_{B,0}$ are the initial position and velocity of the ball. As the ball's flight trajectory was calculated between two contacts, $\dot{x}_{B,0}$ was determined by the impact relation:

$$(\dot{x}_B^+ - \dot{x}_R) = -\alpha(\dot{x}_B^- - \dot{x}_R) \quad (2)$$

where $\dot{x}_B^-$ and $\dot{x}_B^+$ denote the velocity of the ball immediately before and after contact, respectively. $\dot{x}_R$ denotes the velocity of the racket at impact, assuming instantaneous impact. The coefficient of restitution α captures the energy loss at impact. The elastic impact described in Eq. (2) assumed that the mass of the racket was sufficiently larger than the mass of the ball so that the effect of the impact on the racket trajectory could be ignored. $x_{B,0}$ in Eq. (1) was given by the position of the impact measured in extrinsic coordinates.

2.3. Procedure and experimental conditions

The task for the subject was to rhythmically bounce the virtual ball for the duration of each trial such that the peaks of the ball trajectory were as close as possible to the target presented as a horizontal line on the screen (Fig. 2). The target position was about 1 m above the average impact position, measured in extrinsic (screen) space. The trials began with the ball appearing on the left side of the screen and rolling on a horizontal line extending to the middle of the screen. When the ball reached the end of the line, it dropped down towards the racket (Fig. 2). The horizontal position of the racket was fixed and centered under the drop position of the ball. The task of the subject was to bounce the ball so that its peak amplitude reached the target line after every bounce. This starting procedure was designed to visually prepare subjects for the beginning of the task and to introduce the target. After each trial, feedback about the performance was given to the subjects in terms of the mean absolute error, AE, defined as the mean absolute distance between the peak of each ball amplitude and the target.

The experiment comprised two conditions: unperturbed (UP) and perturbed (PE). In the UP condition, the coefficient of restitution ($\alpha = 0.50$) remained constant during the entire trial. In the PE condition, perturbations were introduced by changing α at every fifth impact. That is, α was set to 0.50 over four consecutive impacts but was randomly changed at every fifth impact of the trial. Perturbations were applied at that frequency to obtain a large number of perturbations. They were applied every fifth cycle to have the same number of post-perturbation cycles for statistical analyses. Although the occurrence of a perturbation was in

principle predictable, subjects could not preplan their responses, as the values of perturbed α were randomly specified and some of these perturbations resulted in amplitudes that were even within the range of the self-induced amplitude variation observed for unperturbed trials. To obtain sufficiently large perturbations, the perturbed coefficient of restitution (α_p) was set to be different from its normal value by at least 0.10. To prevent too large perturbations, the maximum difference from the normal value was set to 0.20. Hence, α_p was randomly determined for each perturbed impact such that α_p could be any value within the ranges $0.30 \leq \alpha_p \leq 0.40$ and $0.60 \leq \alpha_p \leq 0.70$, specified to two decimals.

Subjects received two practice trials in each condition. The experiment proper consisted of 15 trials per condition. One trial lasted 40 s and the bouncing frequency was approximately 800–900 ms. Hence, one trial gave approximately 50 cycles and contacts and 10 perturbations for each PE trial. The PE and UP conditions were alternated for every trial. Half of the subjects began with a UP trial while the other half began with a PE trial. The experiment lasted approximately 1 h.

2.4. Data reduction and analyses

The raw data of racket displacements were filtered with a second-order Butterworth filter using a cut-off frequency of 15 Hz.

2.4.1. Description of performance

The error for each cycle (AE) was calculated as the absolute distance between the peak of the ball trajectory and the target height (Fig. 3). The quality of performance was evaluated by the average of the absolute errors across one trial ($\overline{AE}$). Note that for PE, the errors in the cycles

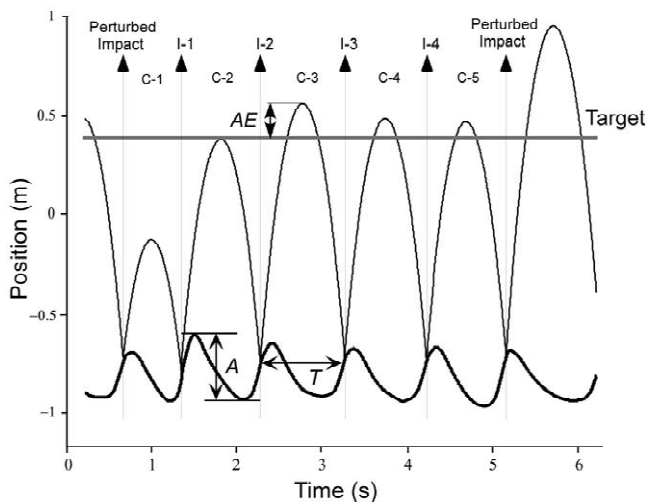


Fig. 3. Illustration of the cycle and impact numbers after the perturbation for an exemplary trial in the PE condition. The calculation of the dependent variables as illustrated in the figure are similarly defined in both the PE and UP.

directly following the perturbed impact at C-1 (AE_1) were removed from this calculation, since these errors could not be attributed to the subject's lack of control.

2.4.2. Acceleration at impact

The acceleration of the racket at impact (AC) was obtained from the racket position data. The filtered racket position data was differentiated, filtered again with the same Butterworth filter (zero-lag, second-order Butterworth filter, cut-off frequency: 15 Hz), and differentiated again to yield the acceleration signal. This procedure was tested and verified by running several trials with an accelerometer on the racket and by comparing the accelerometer data with the acceleration obtained from the differentiated position signal.

2.4.3. Impact phase

The actual phase $\phi_{act,i}$ of the impact i in the racket cycle was calculated as follows:

$$\phi_{act,i} = 2\pi \cdot \frac{t_{l,i} - t_{p,i-1}}{t_{p,i} - t_{p,i-1}} \mod 2\pi \quad (3)$$

where $t_{l,i}$ is the time of the i th impact, and $t_{p,i-1}$ and $t_{p,i}$ are the times of the peak of the racket trajectory before and after the impact i , respectively. $\phi_{act} = 0$ or 2π rad corresponds to an impact at the peak of the racket trajectory. Assuming sinusoidal motion of the racket, ϕ_{act} between $3\pi/2$ and 2π rad denotes an impact during upward motion with negative acceleration of the racket. Note, that racket movements with negative accelerations during downward motion are not a solution, as due to the energy loss at impact, upward momentum needs to be imparted to the ball. To obtain an estimate about how many impacts were performed with and without racket accelerations in the range with passive stability, the number of impacts within a trial outside the range $[3\pi/2, 2\pi \text{ rad}]$ were counted and divided by the total number of impacts per trial and reported as percentages.

To obtain an estimate of the magnitude of the perturbations, we further calculated an 'expected' phase of impact (ϕ_{exp}), assuming that no adjustments were made by the racket. To this end, the time of the next contact following a perturbed impact was calculated by extrapolating the racket trajectory by one average racket period. Subsequently, the phase of this contact time $t_{l,i}$ was determined assuming sinusoidal racket movements:

$$\phi_{exp,i} = 2\pi \cdot \frac{t_{l,i} - t_{p,i-1}}{\bar{P}} \mod 2\pi \quad (4)$$

where $\bar{P}$ is the peak-to-peak period of the racket trajectory averaged over one trial. The contacts with $\phi_{exp,i}$ outside the range $[3\pi/2, 2\pi \text{ rad}]$ indicate that the perturbations would have destabilized the ball–racket pattern, if the racket movements had not been modulated. The number of $\phi_{exp,i}$ that were not in $[3\pi/2, 2\pi \text{ rad}]$ were counted,

divided by the number of contacts in the respective trial, and converted to percent. A non-zero difference between ϕ_{exp} and ϕ_{act} captured that racket modulations were applied to compensate for the perturbations. For an additional check of the robustness of these calculations, the same measures were calculated by using the period P of the cycle before each perturbation, instead of the average period of all cycles of a trial $\bar{P}$. Based on this pre-perturbation cycle period we determined ϕ_{exp} using Eq. (4).

2.4.4. Racket amplitude and cycle time

The movements of the racket were assessed by the racket amplitude (A) and period (T). A was defined as the distance between the minimum and maximum vertical position of the racket during a cycle (Fig. 3). T was defined as the interval between two successive impacts. For each trial, mean values and their standard deviations SDA and SDT were calculated.

2.4.5. Period modulation

For each cycle of the perturbed trials C- i , the modulation of the i th cycle period was calculated by subtracting $\bar{P}$ from the actual periods P_i :

$$P_{\text{mod},i} = P_i - \bar{P} \quad (5)$$

In order to relate $P_{\text{mod},i}$ to the magnitude of the applied perturbation for each cycle number after the perturbation, $P_{\text{mod},i}$ was plotted against α_p and linear regressions were performed.

2.4.6. Covariation between impact parameters (COV)

The degree of error compensation following a perturbation was further evaluated by a covariation measure that analyzed ball and racket kinematics at each contact. Based on the basic mechanical fact that the amplitude of the ball is completely determined by the velocities of racket and ball at contact, the ball amplitude, and consequently, the error AE_i for a given cycle C- i can be calculated from the equations of ballistic flight and elastic impact. Starting with the fact that the kinetic energy immediately after contact is equal to the potential energy at ball amplitude $x_{B,\text{max}}$ we can write:

$$1/2 \cdot m_B (\dot{x}_B^+)^2 = m_B g x_{B,\text{max}} \quad (6)$$

where m_B is the mass of the ball, $\dot{x}_B^+$ is the velocity of the ball immediately after contact, and $x_{B,\text{max}}$ is the peak amplitude of the ball trajectory defined relative to the initial height of the ball-racket contact $x_{B,0}$. $x_{B,\text{max}}$ is thus a function of the initial velocity:

$$x_{B,\text{max}} = (\dot{x}_B^+)^2 / 2g \quad (7)$$

The target height in our experiment, however, is defined in extrinsic space. Thus, it is both the impact positions $x_{B,0}$

and $x_{B,\text{max}}$ that determine the ball's peak position in extrinsic space and, consequently, its deviation from the target height. From Eq. (2), $\dot{x}_B^+$ is a function of the velocity of the racket at contact, $\dot{x}_R$, the velocity of the ball just before impact, $\dot{x}_B^-$, and the coefficient of restitution α . On the other hand, the error E_i following I- i is given by:

$$E_i = x_{B,\text{max},i} + x_{B,0,i} - x_T \quad (8)$$

where x_T is the target height. Taken together, the absolute error AE_i is completely determined at I- i :

$$AE_i = f(x_{B,0,i}, \dot{x}_{R,i}, \dot{x}_{B,i}^-, \alpha, x_T) \quad (9)$$

It is important to note that the same peak height, and thus the same AE_i , can be achieved by different combinations of these parameter values at impact. A high $\dot{x}_R$ at a low $x_{B,0}$ can reach the same height as a small $\dot{x}_R$ at a high $x_{B,0}$. In this case, both parameter combinations are equivalent with respect to the performance. An explicit control strategy may therefore aim to covary these parameter values to achieve the desired target height rather than aim to replicate a given set of contact parameters.

COV can be assessed by a permutation method developed by Müller and co-workers [13–15] (see also Ref. [10]). First, AE_i is computed at each impact from the three impact parameters for each trial and then the average $\overline{AE}$ is determined. To estimate the contribution of covariation between contact parameters at each contact (COV) of each trial, the triplets of parameters are permuted across impacts. For example, the values of $\dot{x}_R$ of all contacts within one trial were permuted, and similarly, the values of all $\dot{x}_B^-$ within one trial were permuted. The new triplets were used to calculate the resulting performance and the absolute error ($AE_{p,i}$). This procedure is illustrated in Table 1 for four impacts. To reduce chance effects, the permutation was performed 10 times and the resulting $AE_{i,p}$ were averaged across all contacts to obtain $\overline{AE}_p$. Finally, the benefit obtained from covariation COV was estimated by subtracting the actual average error from the permuted average error: $\overline{AE}_p - \overline{AE}$.

2.5. Statistical analyses

Each dependent measure $\overline{AE}$, A , T , SDA, SDT, AC,

Table 1

Illustration of the calculation of covariation between impact parameters COV using the permutation method described in Materials and methods

	Measured			AE_i	Permuted			$AE_{p,i}$
	$x_{B,0,i}$	$\dot{x}_{R,i}$	$\dot{x}_{B,i}^-$		$x_{B,0,i}$	$\dot{x}_{R,i}$	$\dot{x}_{B,i}^-$	
I-1	$x_{B,0,1}$	$\dot{x}_{R,1}$	$\dot{x}_{B,1}^-$	AE_1	$x_{B,0,4}$	$\dot{x}_{R,2}$	$\dot{x}_{B,1}^-$	$AE_{p,1}$
I-2	$x_{B,0,2}$	$\dot{x}_{R,2}$	$\dot{x}_{B,2}^-$	AE_2	$x_{B,0,1}$	$\dot{x}_{R,4}$	$\dot{x}_{B,3}^-$	$AE_{p,2}$
I-3	$x_{B,0,3}$	$\dot{x}_{R,3}$	$\dot{x}_{B,3}^-$	AE_3	$x_{B,0,2}$	$\dot{x}_{R,3}$	$\dot{x}_{B,4}^-$	$AE_{p,3}$
I-4	$x_{B,0,4}$	$\dot{x}_{R,4}$	$\dot{x}_{B,4}^-$	AE_4	$x_{B,0,3}$	$\dot{x}_{R,1}$	$\dot{x}_{B,2}^-$	$AE_{p,4}$
				$\overline{AE}$				$\overline{AE}_p$

COV was calculated for each trial and subject and analyzed in two different analyses of variance (ANOVAs). A mixed-design 6 (subject) $\times$ 2 (perturbation) ANOVA was conducted to provide an overall comparison of kinematic measures across the two perturbation conditions. Subject was treated as the between factor, and perturbation as the within factor. With a focus on the PE condition, the dependent variables of each cycle or contact within a trial were sorted and pooled for each cycle or impact number after the perturbation (Fig. 3). The objective was to evaluate how subjects re-established a stable regime after the perturbation. A , T , SDA , SDT were calculated separately for each of the cycles C-1 to C-5. $\overline{AE}$ was also calculated separately, but only for C-2 to C-5. The error in C-1 was not included in the evaluation, as it was the direct result of the perturbation. AC and COV were calculated for all impacts following the perturbation from I-1 to I-4. A one-way repeated-measures ANOVA (either for cycle number or impact number) was performed on the dependent variables. Tukey HSD posthoc tests were performed on all significant effects. For all analyses, only significant results were reported. The level of significance for all analyses was set to 0.01.

3. Results

To evaluate the overall performance in the different conditions a mixed-design 6 $\times$ 2 ANOVA was performed on the trial means $\overline{AE}$. It revealed an interaction between perturbation and subject, $F(5, 84) = 5.07$, $P < 0.01$, a main effect of perturbation, $F(1, 84) = 62.06$, $P < 0.01$, and a main effect of subject, $F(5, 84) = 6.40$, $P < 0.01$. Fig. 4A shows that each subject performed better for UP, despite variations between subjects. The error bars indicate 2 standard errors of the mean. Overall, $\overline{AE}$ was significantly lower for UP ($\overline{AE} = 0.117$ m) than for PE ($\overline{AE} = 0.156$ m). Focusing only on the perturbed condition, the error $\overline{AE}_i$ was compared across the four post-perturbation cycles with a one-way ANOVA. A main effect of cycle number showed that $\overline{AE}_i$ decreased significantly from C-2 to C-5, $F(3, 15) = 6.69$, $P < 0.01$ (Fig. 4B). The subject averages of $\overline{AE}_i$ across C-2 to C-5 were 0.163, 0.182, 0.147 and 0.134 m, respectively. Posthoc pairwise comparisons identified significant differences between C-3 and C-4, and C-3 and C-5. For every cycle, these values remained a little higher than what was obtained in UP (0.117 m) as indicated by the dashed line in Fig. 4B.

The next focus was on the question whether performance was consistent with passive stability. A two-way ANOVA performed on the trial means of AC revealed main effects for perturbation, $F(1, 84) = 7.46$, $P < 0.01$, and for subject, $F(5, 84) = 36.42$, $P < 0.01$. Mean AC is more negative for UP (-2.16 ms $^{-2}$) than for PE (-1.95 ms $^{-2}$). Fig. 5A shows that this was the case for every subject with

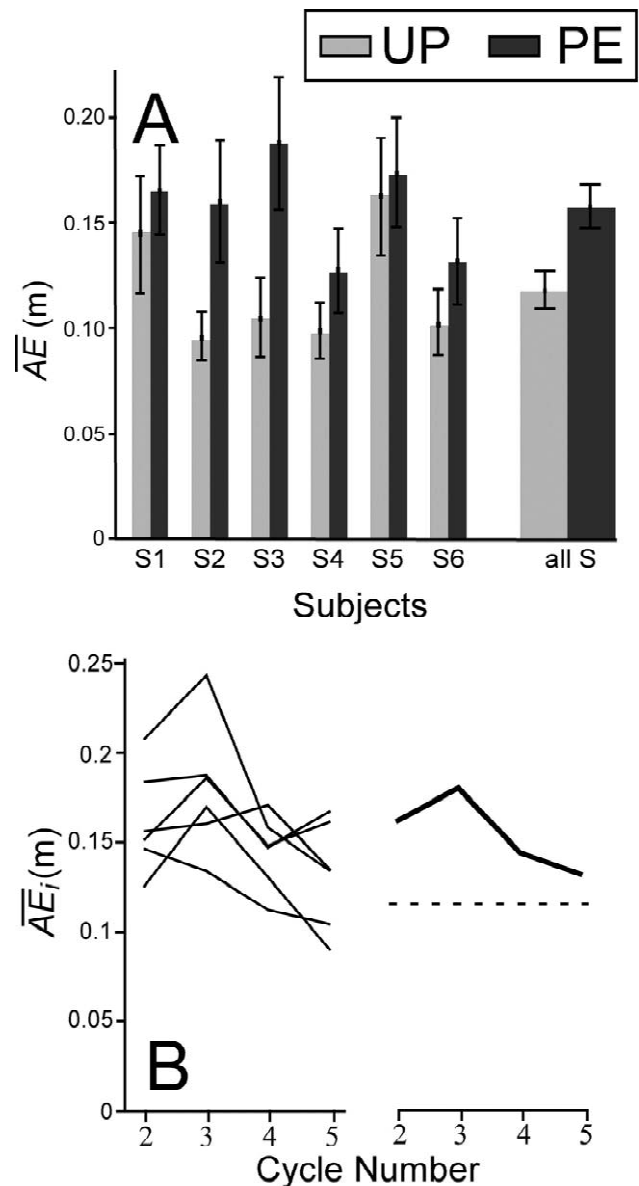


Fig. 4. (A) The average absolute error $\overline{AE}$ for each subject and each perturbation condition, as well as the overall average of $\overline{AE}$ computed over subjects. The error bars indicate ± 2 standard errors of the mean. (B) $\overline{AE}$ obtained in the PE condition as a function of cycle number, shown separately for each subject (left) and averaged over subjects (right). The dotted line corresponds to the baseline level of $\overline{AE}$ obtained in the UP condition.

the exception of subject 5. The one-way ANOVA on AC obtained in PE gave a significant effect of impact number, $F(3, 15) = 49.47$, $P < 0.01$. Fig. 5B reveals that this effect expressed the monotonic decrease in AC from I-1 to I-4: -1.47 , -1.93 , -2.16 , -2.18 ms $^{-2}$, respectively. Posthoc tests identified pairwise differences between I-1 and all others, and I-2 and I-4. At I-3 and I-4 AC came close to the baseline level obtained for UP (-2.16 ms $^{-2}$).

The next set of analyses evaluated the phase of impact with the objective to estimate the degree to which racket movements were modulated due to perturbations to re-

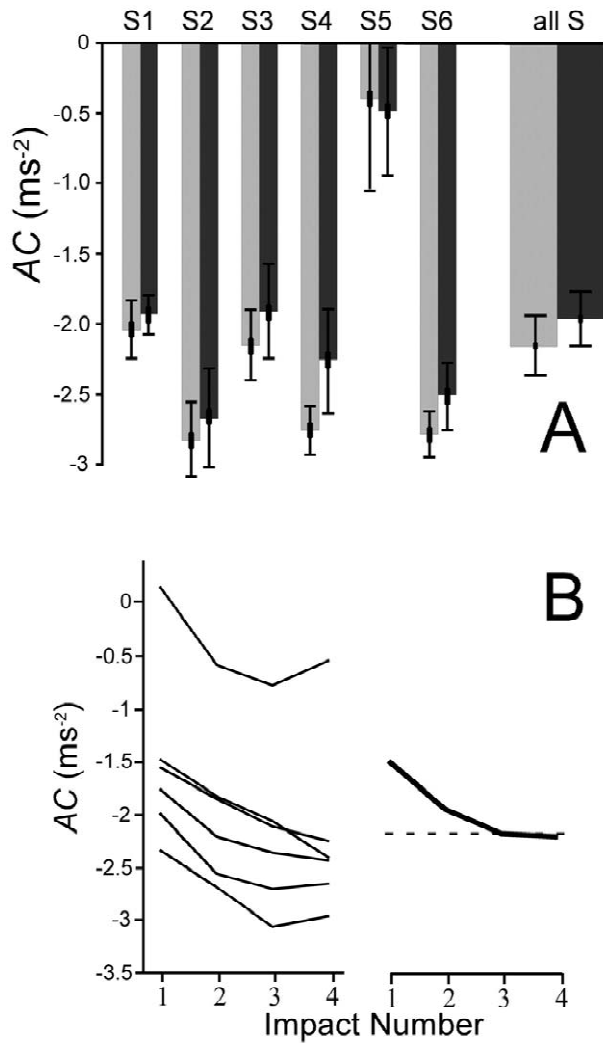


Fig. 5. (A) Mean AC for each subject and each perturbation condition, as well as the subject averages. PE is shown in black, UP is shown in gray. The error bars indicate ± 2 standard errors of the mean. (B) Mean AC of the PE condition presented as a function the cycle number for each subject (left) as well as averaged over subjects (right). The dotted line represents to the baseline level in the UP condition.

establish passive stability. Analyses of ϕ_{act} showed that only 1.6% and 1.8% were outside the range that provided passive stability $[3\pi/2, 2\pi \text{ rad}]$. Fig. 6A and B illustrate this results with histograms for UP and PE where ϕ_{act} of all cycles in all trials and subjects were pooled. By our definition, this meant that almost every impact benefited from passive stability. On the other hand, when extrapolating the previous impact by an average cycle period to compute the expected phase of impact, ϕ_{exp} , 19.3% (UP) and 35.1% (PE) of impacts were outside this range (Fig. 6C and D). These values represent the percentage of impacts in which the racket trajectory needed to be modulated. As expected, this percentage was higher in PE. It was also significantly higher than for ϕ_{act} , which permits the conclusion that racket movements were modulated to ensure impacts with passive stability.

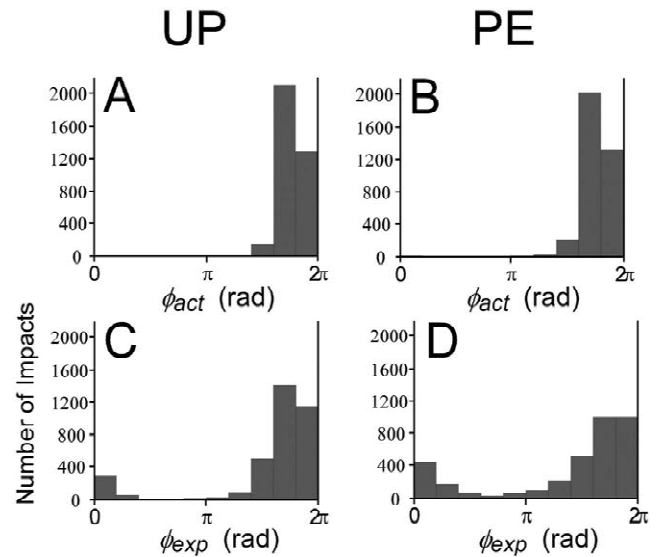


Fig. 6. Histograms for the number of cycles for ϕ_{act} and ϕ_{exp} for UP and PE.

For the data obtained in PE, ϕ_{act} and ϕ_{exp} were also compared across impact number. The histograms in Fig. 7A and B show the number of impacts for ϕ_{act} and ϕ_{exp} in five separate histograms. Note that ϕ_{exp} could also be calculated for a fifth impact, which in reality was the next perturbed impact. The two colors distinguish between perturbations where α_p was higher or lower than α , leading to higher or lower ball amplitudes. For all cycle numbers and all α_p , the impacts occurred between $3\pi/2$ and 2π rad (Fig. 7A). The percentages of ϕ_{act} outside $[3\pi/2, 2\pi \text{ rad}]$ were negligible: 3.1, 3.0, 1.7, 0.6, and 0.2% for I-1 to I-5, respectively. In contrast, the percentages of ϕ_{exp} outside the range were 73.9, 35.4, 28.8, 23.7, and 19.8% for I-1 to I-5, respectively. This meant that in all cycles the racket movements were modulated to obtain a contact phase with negative acceleration. The difference between ϕ_{act} and ϕ_{exp} was much higher in I-1, and decreased towards I-5. The separate depiction of impacts with higher (gray) and lower (black) α_p revealed that ϕ_{exp} following from lower α_p were mainly between π and $3\pi/2$ rad in I-1. This means that the impacts would have occurred earlier if the racket period had not been modulated. Conversely, ϕ_{exp} that followed from higher α_p were mainly between 0 and π rad in I-1, which corresponded to an impact occurring at an early phase of the next cycle if the racket period had not been modulated¹.

Fig. 7C shows continuous racket trajectories of one subject that illustrates how the trajectories were modulated.

¹When the same measures were calculated using the one cycle period before the perturbation, the same pattern of distributions was obtained for all cycles. The percentages differed maximally by 5% for the different impacts. The results presented above had a tendency to show slightly lower percentages for impacts that demonstrate adjustments. The percentages for ϕ_{exp} are: 73.5, 39.9, 32.7, 28.5, 24.7%.

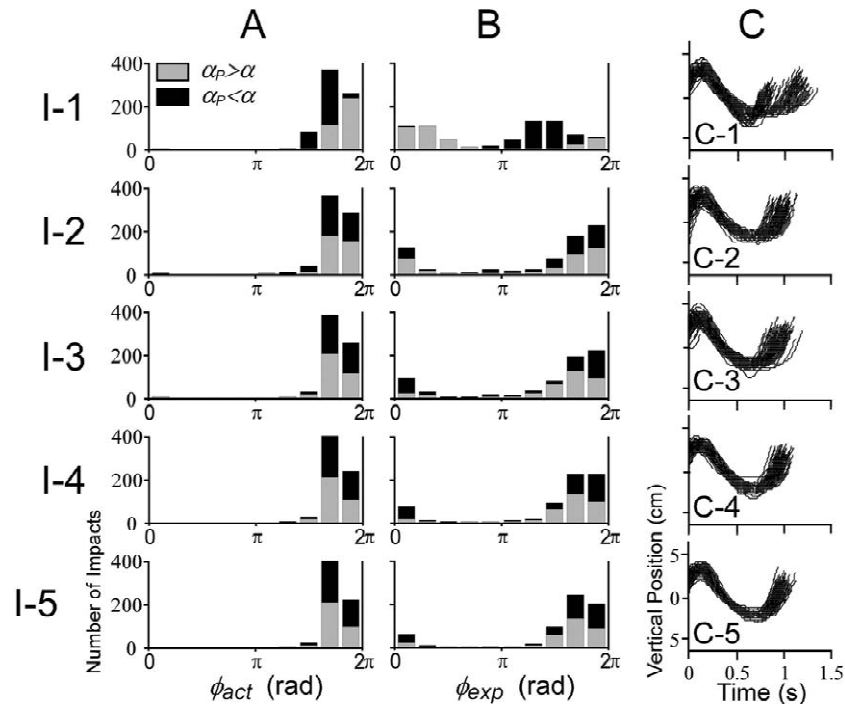


Fig. 7. (A) and (B) Histograms of ϕ_{act} and ϕ_{exp} obtained in PE are presented as a function of the cycle number. Impact phase following perturbation with $\alpha_p > \alpha$ are presented in gray, while perturbations with $\alpha_p < \alpha$ are presented in black. (C) Continuous racket trajectories of one subject parsed into cycles and sorted for cycle number.

The trajectories of all 15 trials were parsed into individual cycles that were sorted according to cycle number as was done for other dependent measures above. To eliminate vertical drifts in the racket movement during the trial, the racket cycles were centered around zero. For C-1 one can distinguish the two patterns of racket trajectories: for high α_p the trajectory periods were lengthened, for low α_p the periods were shortened. In C-2 to C-5, the two patterns can no longer be distinguished.

To further elucidate the modulation of the racket trajectories the racket kinematics for the different conditions were evaluated by their mean amplitude A and mean period T and its standard deviations SDA and SDT. The mean values of A and T for each subject and each condition are presented in Table 2. Two 6×2 ANOVAs performed on A and T revealed significant main effects

Table 2
Means of six subjects' real racket amplitude A (cm) and racket cycle time T (s) for the two experimental conditions (unperturbed, UP and perturbed, PE)

	A		T	
	UP	PE	UP	PE
S1	4.68	4.94	0.85	0.91
S2	4.69	5.02	0.81	0.83
S3	5.96	6.67	0.89	0.90
S4	3.55	3.69	0.84	0.85
S5	4.22	4.06	0.90	0.91
S6	5.41	5.51	0.96	0.98

only for subject: A: $F(1, 84)=58.87$; T : $F(1, 84)=12.78$. The two-way ANOVA for SDA revealed a main effect for subject, $F(5, 84)=5.46$, $P<0.01$, and a main effect for perturbation, $F(1, 84)=9.45$, $P<0.01$. SDA was significantly higher for PE (1.55 cm) than for UP (1.34 cm) (Fig. 8A). This difference was also expressed as percent of UP that represented the baseline level of variability in the unperturbed task performance. SDA increased by 16% from UP to PE. Analyzing variability across cycles in PE with a one-way ANOVA did not render significant differences. Fig. 8B illustrates the individual subjects' patterns in SDA as a function of cycle number. While no systematic pattern across subjects can be distinguished, the average SDA remained slightly higher in PE (0.016, 0.018, 0.018, 0.016, 0.015 m, for C-1 to C-5, respectively) compared to UP (0.0134 m).

Turning to SDT, the mixed-design ANOVA revealed only a main effect of perturbation, $F(1, 84)=165.35$, $P<0.01$. SDT was higher for PE (0.123 s) than for UP (0.070 s). Fig. 8C shows that this was consistently seen in every subject. The increase of SDT from PE to UP amounted to 76% of the SDT obtained in UP. This is considerably higher than the 16% increase obtained for SDA. The separate analysis of the PE trials found a significant effect of cycle number, $F(4, 20)=53.30$, $P<0.01$. Fig. 8D revealed the marked decrease in SDT across all cycles following the perturbation. From C-1 to C-5: 0.195, 0.117, 0.123, 0.086, 0.078 s. All pairwise posthoc comparisons were significant with the exception of C-2

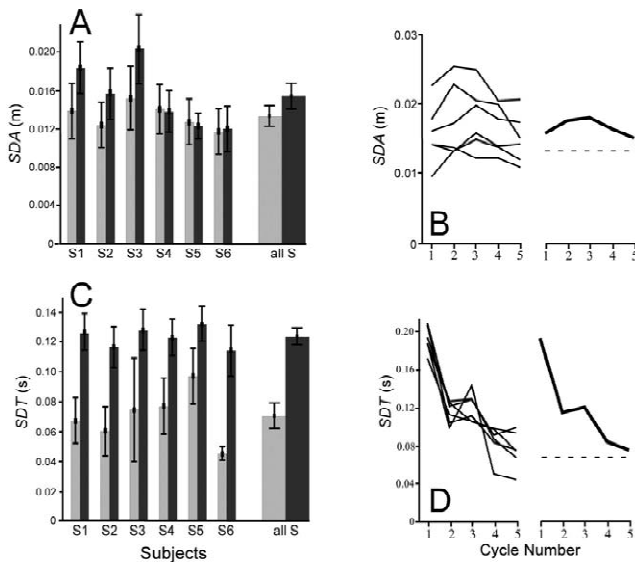


Fig. 8. (A) and (B) SDA and SDT for each subject and each perturbation condition, as well as the average over subjects. The error bars indicate ± 2 standard errors of the mean. (C) and (D) SDA and SDT obtained in the PE condition as a function of the cycle number for each subject (left) and averaged over subjects (right). The dotted line represents the baseline level in the UP condition.

with C-3 and C-4 with C-5. This effect was consistent among subjects. While SDT was high in C-1, it returned close to baseline in C-5.

In order to relate the modulation of the racket period to the magnitude of the applied perturbation, $P_{mod,i}$ were calculated for each cycle number after the perturbation and regressed against α_p . Fig. 9 shows these plots separately for the five cycles. For C-1, a clear relation appears: the racket period was shortened for α_p lower than 0.5 and lengthened for α_p higher than 0.5. Linear regressions revealed a significant correlation for C-1, $r=0.80$, $P<0.01$. When linear regressions were performed separately for the data obtained with $\alpha_p<0.5$ and $\alpha_p>0.5$ the correlations were also significant, $r=0.24$, $P<0.01$ and $r=0.23$, $P<0.01$, for $\alpha_p<0.5$ and $\alpha_p>0.5$, respectively. For C-2 to C-5, only marginal relations appeared. Linear regressions revealed that these relations were also above the level of significance for C-2 to C-5: $r=0.18$, $r=0.19$, $r=0.23$, all P values <0.01 . However, when linear regressions were performed separately for data obtained with $\alpha_p<0.5$ and $\alpha_p>0.5$, none of these correlations were significant.

Lastly, covariation COV was assessed as a measure that captured error compensation between parameters at im-

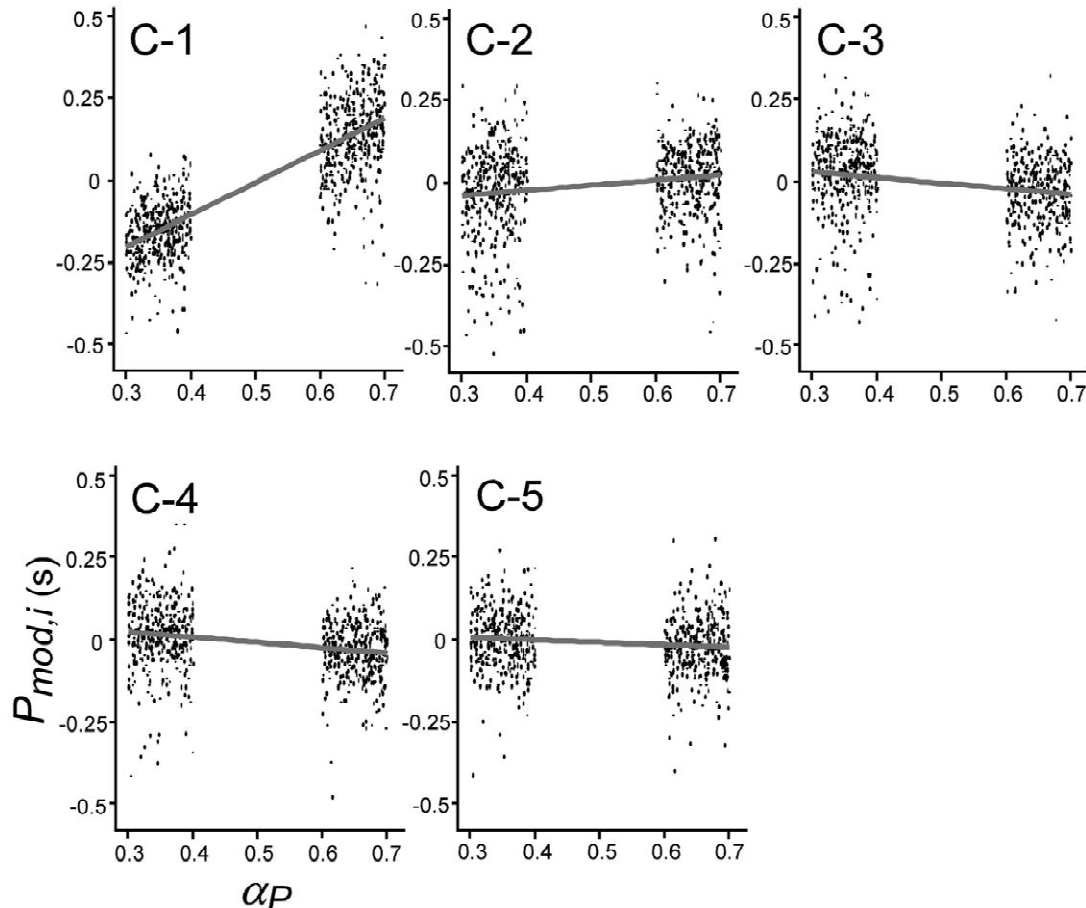


Fig. 9. $P_{mod,i}$ plotted against α_p for each cycle number after the perturbation, from C-1 to C-5. The gray lines represent the linear regressions.

pact. The 2×6 ANOVA performed on COV revealed a main effect of perturbation, $F(1, 84) = 164.89$, $P < 0.01$. The improvement in performance as measured in AE due to COV is more than doubled from UP (0.032 m) to PE (0.067 m). Fig. 10A illustrates this pronounced difference and its consistency among subjects. A one-way ANOVA on COV obtained in PE also revealed an effect of impact number, $F(3, 15) = 50.09$, $P < 0.01$. Fig. 10B illustrates that benefits of the performance from COV is much higher for I-1 (0.147 m) and drops to approximately baseline level for UP (0.032 m) on the I-2 to I-4: 0.039, 0.048, 0.034 m, respectively. Posthoc tests mark significance for the difference between I-4 and all others.

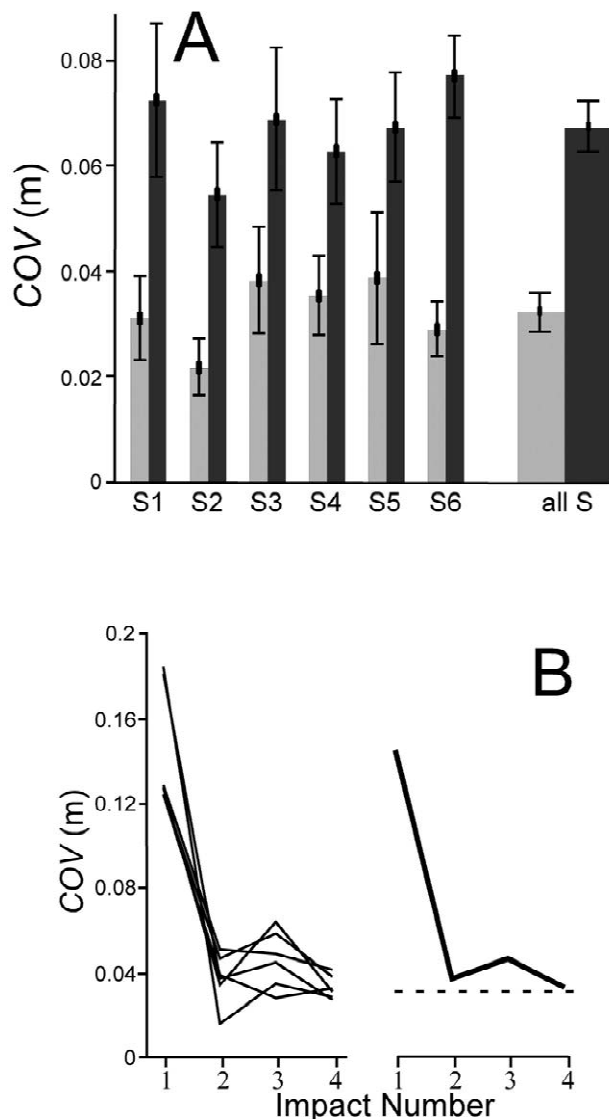


Fig. 10. (A) COV for each subject and each perturbation condition, as well as the average over subjects. The error bars indicate ± 2 standard errors of the mean. (B) COV obtained in the PE condition as a function of the cycle number for each subject (left) and subject averages (right). The dotted line represents the baseline level in the UP condition.

4. Discussion

Bouncing a ball regularly on a racket is a rhythmic task that involves the intricate coordination between the movements of the racket and the movements of a ball. Two types of control have been suggested to perform this skill. The first strategy follows classical control theory where the racket trajectory is planned and continuously controlled on the basis of visual feedback from the movement of the ball. Koditschek and colleagues applied such type of control algorithm for the control of a juggling robot [6,16]. The second strategy makes use of the stability properties that the task offers and allows for open-loop control as small perturbations converge back to the stable state [7,20,21]. Motivated by this model, human experiments on steady state performance of bouncing a ball suggested that humans employ the second strategy and attune to this passive stability [9,17,19,20]. This conclusion was based on the fact that their racket trajectory was decelerating at ball-contact, as predicted by stability analyses for dynamically stable solutions. Even though these results gave evidence that dynamical stability plays a major role in subjects' coordination, they do not rule out that additional perception-based control was also applied. Such feedback-based control is needed to establish the steady state regime or when large perturbations occur that have to be compensated for. Further, it may even exist when passive stability is present. The objective of the present study was to identify the control mechanisms involved when large perturbations had to be counteracted.

4.1. Experimental results

As this experiment was the first one run with the virtual set-up, it was necessary to establish that dynamical stability was present in the unperturbed conditions as shown in previous studies with real ball racket interactions [19,20]. All subjects performed consistently with negative accelerations of the racket at impact in both UP and PE confirming previous results that passive stability played a role in their coordination. This provided the starting point from which the effect of perturbations could be studied. Another prerequisite was that even in the perturbed conditions performance was still 'under control', in the sense that subjects did not lose the bouncing pattern completely. The evaluation of errors with respect to the target showed that while performance was expectedly poorer in the perturbed condition, performance did not drop dramatically from UP to PE and a reasonable level was still maintained in PE. Overall, steady state values of kinematic descriptors were reestablished after approximately two to three post-perturbation cycles, suggesting that the perturbations were compensated for. However, this finding cannot distinguish whether this stabilization was due to the 'passive' stability properties, as illustrated in Fig. 1, or due to active error compensation.

Analyses of the impact phase revealed that an explicit error compensation strategy was used. The phases of the impacts calculated from the actual racket trajectory and from an extrapolated racket period strongly support the operation of a control mechanism that timed the ball-contacts to a phase region that provided dynamical stability. With the exception of a small percentage of impacts, the actual phases of impact showed negative accelerations. On the other hand, the expected impact phases showed that a marked proportion of impacts were out of the range $[3\pi/2, 2\pi \text{ rad}]$, indicating that they needed a modulation of the racket period to benefit from passive stability. This proportion was higher in PE than in UP, although it was not negligible in UP (35.1% in PE versus 19.3% in UP). This meant that even without perturbations, the racket trajectory was apparently modulated. Phase results as a function of the cycle number further indicated that such modulation was most pronounced during the first post-perturbation cycle, and gradually reached the baseline level of UP at the fifth cycle. Note that these calculations were based on an idealized range of a quarter cycle. If the racket trajectories deviated from smooth sinusoidal movements, then errors in our calculations were present. However, in conjunction with the mean results on negative racket acceleration, these phase results appear sufficiently reliable.

Another sign that racket movements were modulated in the first post-perturbation cycles was the significant increase in variability of the racket amplitude and period in PE. Since passive stability assumes no modulation of the racket trajectory, this increased variability was an indication of extra adaptations, provided that this variability was not only a uniform increase across all cycles. Comparing estimates at the different cycles, period variability was much higher in C-1, and decreased from C-2 to C-5 toward the baseline level obtained in UP, supporting the above interpretation that the overall variability increase was not only a higher level of random fluctuations. It is important to highlight that this increase in variability from UP to PE was much more frequent for the racket period (76%) as compared to racket amplitude (16%). The slight increase in the variability of the racket amplitude, unspecific to cycle number, appeared more likely to be a byproduct of this control. Another piece of evidence that timing was explicitly modulated in the first cycle after the perturbation was that the magnitude of the period adjustments correlated strongly with the magnitude of the perturbations.

Support for additional control was also the observed covariation between the kinematic parameters at impact, impact height, racket velocity at impact, and ball velocity before impact. Covariation was found to be approximately twice as high in PE compared to UP. Furthermore, this difference was due to a high level of covariation at the first impact after perturbation which leveled out to the level seen in UP on the following impacts. It must also be recognized that covariation is at the core of passive stability as was exemplified in the introductory example

with Fig. 1. Yet, together with the results obtained in impact phase and in the variability of the racket period, covariation highlights that explicit control was applied directly after the perturbation.

In sum, the system recovers from large perturbations using a control mechanism that regulates the timing of successive impacts. This control ensures that ball–racket contacts occur during the upward decelerating phase of the racket trajectory. The same indicators for active control were also seen after stability has been reestablished in the perturbed trials as well as in the unperturbed trials. This suggests that perception-based modulation is also present when the system is in the stable regime, albeit to a lesser degree. These results were implemented in the following model.

4.2. Model

The model system consists of a ‘neural’ oscillator that drives a mechanical limb, mimicking the forearm that bounces a ball with a held racket (Fig. 11). As in the simulations for the virtual set-up, the movement of the ball between two impacts was simulated from the equations of ballistic flight (Eq. (1)). Its initial velocity after impact was calculated from the equation of elastic impact (Eq. (2)).

The oscillator model was developed by Matsuoka expressing the basic mechanism of a half-center oscillator [11,12]. It consists of two neurons (i and j) whose activity is generated by the following equations:

$$\begin{aligned} t_1 \dot{\psi}_i &= -\psi_i + s - b\varphi_i - w[\psi_j]^+ \\ t_2 \dot{\varphi}_i &= -\varphi_i + [\psi_i]^+ \end{aligned} \quad (10)$$

where ψ_i represents the firing rate of neuron i , and φ_i is its self-inhibition. Each neuron receives a tonic input s , and inhibits the other through $-w[\psi_j]^+$ (see Fig. 11). The bracket notation expresses that only positive values are considered and the term is zero if the argument x is negative: $[x]^+ = \max(x, 0)$. w is the gain for this coupling term and b is the gain for the self-inhibition. The tonic input s determines the amplitude of the output, which is a

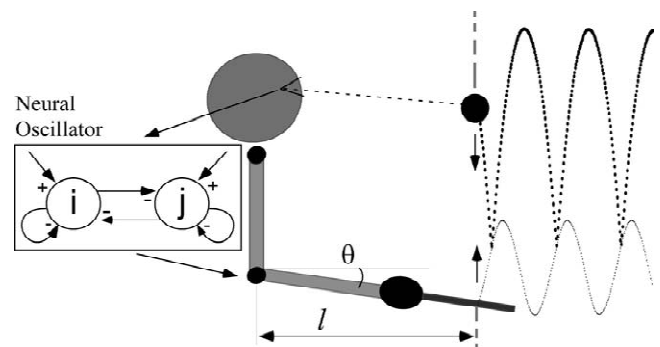


Fig. 11. Illustration of the ball-bouncing model.

decaying burst for each neuron. The two time constants t_1 and t_2 determine its frequency.

The oscillator controls the activity of two antagonistic ‘muscles’ i and j operating at the elbow (Fig. 11). The opposing torques are a linear function of the output of each neuron:

$$\begin{aligned} T_i &= h_T[\psi_i]^+ \\ T_j &= -h_T[\psi_j]^+ \end{aligned} \quad (11)$$

where h_T is the gain of the torques. Angular displacements of forearm and racket θ are generated by the inertial system driven by the imbalance of the two torques:

$$I\ddot{\theta} + \gamma\dot{\theta} + k(\theta - \theta_0) = T_i + T_j \quad (12)$$

where I is the moment of inertia of the forearm plus racket, γ is damping, and k is stiffness of the elbow associated with a rest position at $\theta_0 = 0$ rad. For simplicity, we assumed that the torque associated with gravity is constant, as the range of θ is small, and is compensated for by a constant torque generated by the elbow flexor. The vertical position and velocity of the racket at impact x_R and $\dot{x}_R$ are calculated from θ and $\dot{\theta}$ by the trigonometric relations:

$$\begin{aligned} x_R &= l \tan \theta \\ \dot{x}_R &= l\dot{\theta}(\sec \theta)^2 \end{aligned} \quad (13)$$

where l is the horizontal distance between the elbow joint and the vertical path of the ball (Fig. 11).

The first simulations were conducted to illustrate that this model can perform the ball bouncing movements with passive stability countering small perturbations (Fig. 1). Note that no additional control algorithm has been included yet. The model parameters were chosen so that the same quantitative output was obtained as in the experimental performance. First, the values for I , γ , and k were taken from the literature that measured human rhythmic elbow movements: $\gamma = 0.5$ Nm/rad/s; $k = 5$ Nm/rad; $I = 0.1$ Nm/rad/s² [5]. The value for I was higher to include the inertia added by the racket (estimates in [5] are $I = 0.08$ Nm/rad/s²). Second, the time constants of the oscillator was set to obtain the same target amplitude and period as in the experiment: $P_t = 0.9$ s:

$$\begin{aligned} t_1 &= P_t \times 0.1 \\ t_2 &= P_t \times 0.25 \end{aligned} \quad (14)$$

The oscillator parameters were set to $s = 2$; $b = 2.5$; $w = 2.5$. Third, to obtain P_t associated with a given ball amplitude, racket velocity at impact had to have a given value that, importantly, should occur in the range of negative acceleration. These constraints determined the choice of the gain h_T for the torques, which was set to $h_T = 1.8$. With this parameterization and $\alpha = 0.50$, the system displayed passive stability as illustrated in Fig. 1. For the two ‘small’ perturbed impacts, α_p was set to 0.47 and 0.53.

Fig. 12 demonstrates a series of simulation runs that implemented perturbations with four α_p values taken from the same range as used in the experiment. As can be seen, perturbations with $\alpha_p = 0.7$ revealed that passive stability alone did not maintain a stable bouncing pattern. If the experimental protocol was simulated with perturbations at every fifth impact, stability should also have been lost for $\alpha_p = 0.3$, as recovery from the perturbation took more than five cycles. Hence, a control algorithm was designed that maintained the bouncing pattern for such perturbations as observed in the experiment. To maintain the system in the range of passive stability, a control algorithm could for instance modify the period of the racket to equal that of the ball. Assuming that ball velocity after impact ($\dot{x}_B^+$) can be perceived, the period to the next ball–racket contact, P_a can be anticipated:

$$P_a = \frac{2\dot{x}_B^+}{g} \quad (15)$$

Based on this anticipated period P_a , the oscillator period

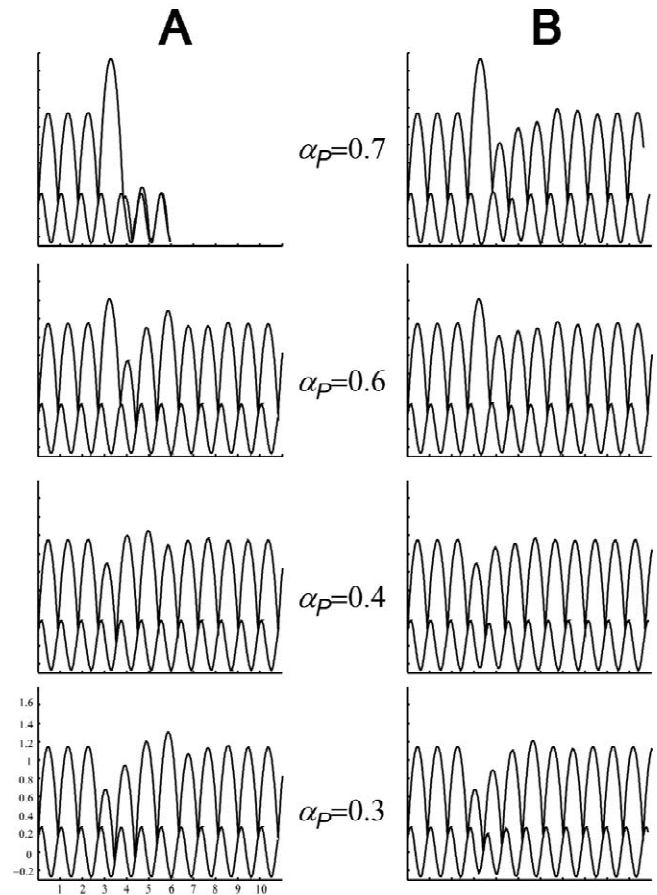


Fig. 12. (A) Simulations performed with the neural–mechanical system without period controller. (B) Simulations performed with the neural–mechanical system with period controller. The applied perturbations were in the same range as in the experiment ($\alpha_p = 0.7$, 0.6, 0.4, and 0.3 on an impact for each simulation, respectively).

P can be reset after each impact to ensure an appropriate phase of ball–racket contact. However, with this type of control, the period shifts away from the target period, thereby violates the task, and stability may eventually be lost. Hence, the objective is to stabilize the system to the target period (P_t). On the basis of P_a convergence toward P_t was implemented by resetting the period of the oscillator to the period P :

$$\begin{cases} P = P_t & |P_a - P_t| \leq \delta \\ P = P_a - \beta(P_a - P_t) & |P_a - P_t| > \delta \end{cases} \quad (16)$$

where δ is a perceptive threshold applied to the difference between P_t and P_a that activates period modulation. β represents the gain for the target period. We refer to this control as the period controller. Small perturbations were below δ and could only be compensated for by passive stability. Larger perturbations were perceived as above δ and were therefore actively compensated for by the period controller that reestablished the conditions for passive stability.

Fig. 12 illustrates simulations without and with the period controller (Fig. 12A and B, respectively), applying perturbations in the same range as in the experiment ($\alpha_p = 0.7, 0.6, 0.4$, and 0.3 on one impact for each simulation, respectively). The parameters of the period controller were $\delta = 0.05$ s and $\beta = 0.2$. As can be seen, stability could be recovered ‘passively’ when perturbations were: $0.3 \leq \alpha_p \leq 0.6$. However, the ball-bouncing system could not recover stability without period control for a perturbation with $\alpha_p = 0.7$. Stability was also recovered more quickly with the period controller for all α_p . Without control, it could take five and more cycles to recover an invariant ball bouncing pattern. When applying repeated perturbations as in the experimental protocol, this led to loss of stability. With the period controller bouncing behavior could be maintained during simulations of the experimental protocol.

A set of simulations was run with the period controller for combinations of the parameters δ and β to simulate the experimental protocol and to compare the simulated results with the ones of the experiment. $\beta = 0$ meant no influence of the target period in the control and could not achieve the task. On the other hand, $\beta = 1$ meant no adjustments since the period of the oscillator was kept constant at the target period. Four different values of β between 0 and 1 were tested: $\beta = 0.2, 0.4, 0.6, 0.8$. To obtain a range of meaningful values for δ , the maximal difference between P_t and P_a that resulted from a perturbation was calculated to be 0.26 s. This meant that for $\delta = 0.26$ s, no adjustments were applied since no deviations were perceived. Four values of δ below 0.26 s were tested: $\delta = 0, 0.065, 0.13, 0.195$ s. For each of the 16 δ – β pairs, simulations were performed with 1000 consecutive impacts, and the dependent measures were calculated from the simulation runs as in the experiment.

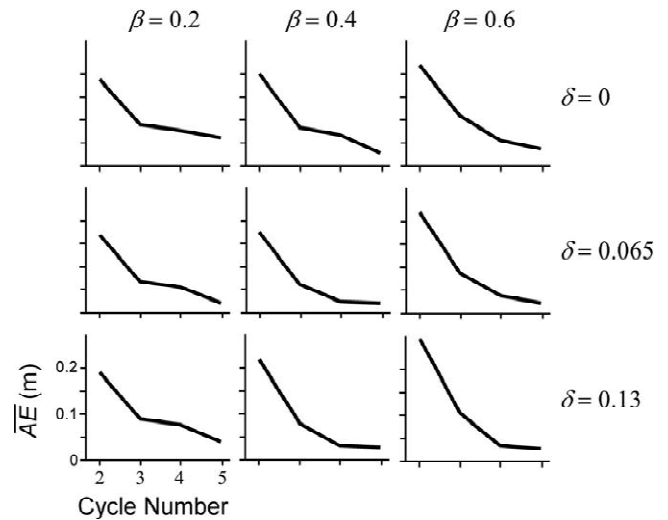


Fig. 13. Simulation results of $\overline{AE}$ as a function of cycle number, using the experimental protocol for 1000 impacts with different values of parameter combinations of δ and β for which stable bouncing was maintained during the 1000 impacts. This criterion was achieved for each combination with $\beta = 0.2, 0.4, 0.6$, and with $\delta = 0.065, 0.13, 0.195$ s.

Figs. 13–16 present the results for $\overline{AE}$, SDA, SDT, and COV, respectively. All simulations run with $\beta = 0.8$ led to unstable bouncing and a complete loss of the pattern, showing that a minimum influence of P_a was necessary. All simulations run with $\delta = 0.195$ s led to unstable bouncing showing that a minimum perceptive threshold is required. The simulated results shown in Figs. 13–16 were performed with all nine successful δ – β combinations for which stable bouncing was maintained throughout 1000 impacts ($\beta = 0.2, 0.4, 0.6$, and $\delta = 0, 0.065, 0.13$ s).

Fig. 13 shows the simulated $\overline{AE}$ that were in the same range as in the experiment and showed the same trend to decrease as a function of the cycle number after the perturbation. Simulated SDA was slightly different from

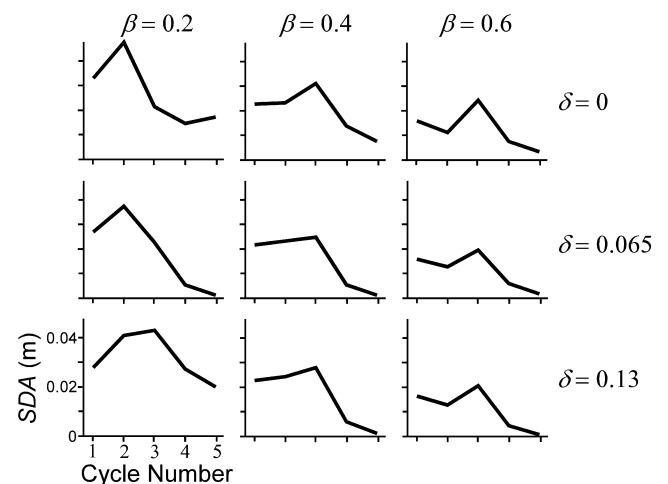


Fig. 14. Simulation results of SDA as a function of cycle number, using the experimental protocol for 1000 impacts with different values of δ and β .

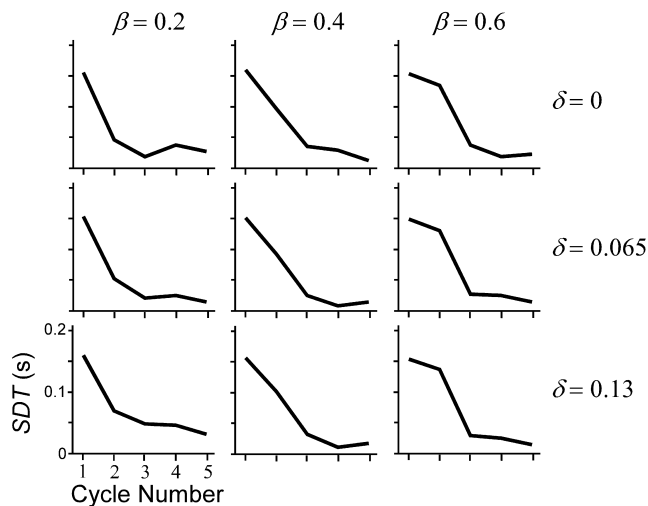


Fig. 15. Simulation results of SDT as a function of cycle number, using the experimental protocol for 1000 successive impacts with different values of δ and β .

the experimental data and showed a tendency to decrease with cycle number. However, as in the experimental data, SDA peaked systematically in the second or third cycle after the perturbation. This was due to the fact that amplitude modification was a byproduct of the period modulation of the oscillator, as it intervened mainly in the first post-perturbation cycle. Due to the neuro-mechanical delay in the model the consequences on the racket amplitudes became overt only in the following cycles.

Simulated SDT were similar to the experimental data, with highest variability on the first cycle, followed by a sharp decrease in the following cycle (Fig. 15). Both the simulated and empirical COV results had a peak in the first cycle, followed by small values in the remaining cycles

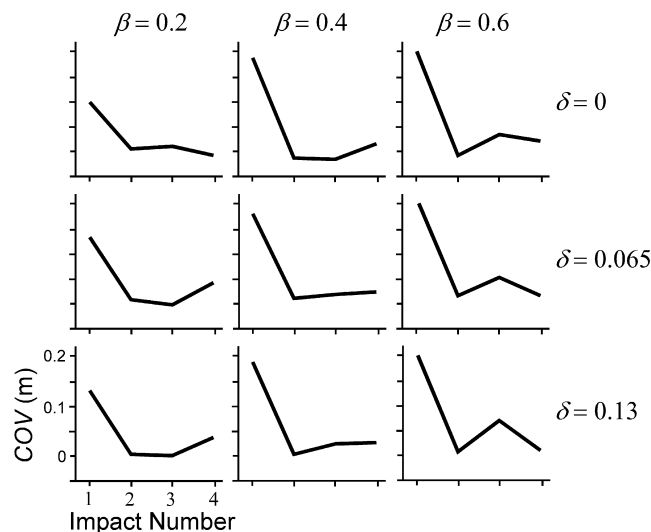


Fig. 16. Simulation results of COV as a function of cycle number, using the experimental protocol for 1000 successive impacts with different values of δ and β .

(Fig. 16). Taken together, these simulated results showed that a range of combinations of δ and β reproduced the major aspects of the experimental data.

In sum, the experimental results and the modeling of the ball bouncing task demonstrated: (1) an invariant oscillatory movement can perform the task with passive stability compensating for sufficiently small perturbations, i.e., no change in racket trajectory is required. In such case, the limb can oscillate and bounce the ball in an open-loop fashion. (2) Additional control is required for larger perturbations that lead to contacts outside the region of negative acceleration. Such control can be implemented by modifying the oscillator period after a perturbed impact. (3) In the model a perceptive threshold was introduced that scaled the influence of the controller based on the difference between anticipated and target period. Note, though, that the controller was monitoring in a continuous fashion. It only implemented a period change when a deviation was above the perceptual threshold. Thus, the model implemented a co-existence between open-loop and closed-loop control: for small perturbations no control was active; for large perturbations explicit corrections were introduced. The period controller only had two parameters with a bounded range. The behavior in this entire parameter space could be tested. Given the quantitative matching between human and model performance, results could be directly compared. It demonstrated that with a simple actuator model with a controller the major results of the experiment could be reproduced. As such, the system ‘tracks’ passive stability with active control of the oscillatory period.

Acknowledgements

This research was supported by a grant from the National Science Foundation, Behavioral Neuroscience 00-96543 awarded to D.S.

References

- [1] E.W. Aboaf, C.G. Atkeson, D.J. Reinkensmeyer, Task-level robot learning. Paper presented at the Proceedings of the IEEE International Conference on Robotics and Automation, Philadelphia, PA, 1988.
- [2] P.J. Beek (Ed.), Juggling Dynamics, Free University Press, Amsterdam, 1989.
- [3] P.J. Beek, W.J. Beek, Tools for constructing dynamical models of rhythmic movement, *Hum. Movement Sci.* 7 (1988) 301–342.
- [4] P.J. Beek, A.A.M. van Santvoord, Dexterity in cascade juggling, in: M.L. Latash, M.T. Turvey (Eds.), *Dexterity and Its Development*, Erlbaum, Mahwah, 1996, pp. 377–392.
- [5] D.J. Bennett, J.M. Hollerbach, Y. Xu, I.W. Hunter, Time-varying stiffness of voluntary elbow joint during cyclic voluntary movement, *Exp. Brain Res.* 88 (1992) 433–442.
- [6] M. Bühler, D.E. Koditschek, P.J. Kindlmann, Planning and control

- of robotic juggling and catching tasks, *Int. J. Robotics Res.* 13 (1994) 101–118.
- [7] J. Guckenheimer, P. Holmes (Eds.), *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer, New York, 1983.
- [8] P.J. Holmes, The dynamics of repeated impacts with sinusoidally vibrating table, *J. Sound Vibr.* 84 (1982) 173–189.
- [9] H. Katsumata, V. Zatsiorsky, D. Sternad, Control of ball–racket interactions in the rhythmic propulsion of elastic and non-elastic balls, *Exp. Brain Res.* 149 (2003) 17–29.
- [10] K. Kudo, S. Tsutsui, T. Ishikura, T. Ito, Y. Yamamoto, Compensatory correlation of release parameters in a throwing task, *J. Motor Behav.* 32 (2000) 337–345.
- [11] K. Matsuoka, Mechanisms of frequency and pattern control in the neural rhythm generators, *Biol. Cybern.* 56 (1987) 345–353.
- [12] K. Matsuoka, Sustained oscillations generated by mutually inhibiting neurons with adaptation, *Biol. Cybern.* 52 (1985) 367–376.
- [13] H. Müller (Ed.), *Ausführungsvariabilität und Ergebniskonstanz*, Pabst Science Publishers, Lengerich, 2001.
- [14] H. Müller, D. Sternad, A randomization method for the calculation of covariation in multiple nonlinear relations: illustrated at the example of goal-directed movements. *Biol. Cybern.*, in press.
- [15] H. Müller, D. Sternad, Decomposition of variability in the execution of goal-oriented tasks—three components of skill improvement. *J. Exp. Psychol. Hum. Percept. Perform.*, submitted for publication.
- [16] A.A. Rizzi, D.E. Koditschek, Progress in spatial robot juggling in: *Proceedings IEEE International Conference on Robotics and Automation*, 1992, pp. 775–780.
- [17] S. Schaal, D. Sternad, C.G. Atkeson, One-handed juggling: a dynamical approach to a rhythmic movement task, *J. Motor. Behav.* 28 (1996) 165–183.
- [18] D. Sternad, Juggling and bouncing balls: parallels and differences in dynamic concepts and tools, *Int. J. Sport Psychol.* 30 (1999) 462–489.
- [19] D. Sternad, M. Duarte, H. Katsumata, S. Schaal, Dynamics of a bouncing ball in human performance. *Phys. Rev. E* 63 (2000) 011902-1–011902-8.
- [20] D. Sternad, M. Duarte, H. Katsumata, S. Schaal, Bouncing a ball: tuning into dynamic stability, *J. Exp. Psychol. Hum. Percept. Perform.* 27 (2001) 1163–1184.
- [21] N.B. Tufillaro, T. Abbott, J. Reilly (Eds.), *An Experimental Approach to Nonlinear Dynamics and Chaos*, Addison-Wesley, Redwood City, CA, 1992.
- [22] L.A. Wood, K.P. Byrne, Analysis of a random repeated impact process, *J. Sound Vibr.* 78 (1981) 329–345.